A Grammatical Evolution Model for Reservoir Inflow Forecasting

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Abstract

This paper explores the feasibility of applying a grammatical evolution (GE) system and combines it with the genetic algorithm (GA) to establish the inflow predicting model of De-Chi Reservoir in central Taiwan. First, a GE is an evolutionary automatic programming type system, which can discover relationships among observed data and express them mathematically. Further, a GA was used with this GE to optimize the appropriate function type automatically. We apply this GE model to fit to the inflow data series on three different conditions, the dry, wet and normal year. Experimental results are presented to demonstrate the applicability of GE for forecasting long-term time series, and the results are found to be better compared with the traditional multi-regressive (MR) method.

Keywords: grammatical evolution, genetic algorithm, inflow predicting model

1. Introduction

Forecasting a historical time series has been of the most complicated tasks owing to the wide range of data. Modeling the inflow process of a reservoir is however inherently complex, highly nonlinear and temporally and spatially non-uniform. Several kinds of data mining techniques are developed, such as statistics, memory-based reasoning, artificial neural networks (ANNs), and decision trees. Evolutionary algorithms have been used with much success for the automatic generation of programs. It has an advantage over traditional statistical methods because it is distribution free, i.e., no prior knowledge is needed about the statistical distribution of the data. However, constructing the data structure of dynamic tree of the genetic programming could be a difficult task while programming [1].

The inflow forecasting model of De-Chi Reservoir in central Taiwan was constructed and discussed in this paper. This data structure of binary string is convenient to combine with the genetic algorithm (GA), which can optimize the functions generated by GE automatically. In the case study, multi-regressive (MR) method and GE approach were used to model reservoir inflows and the results of both were compared through four criteria and three hydrological conditions.
2. Grammatical Evolution

Grammar evolution (GE) has been applied to all manner of automatic programming problems, from symbolic regression, to C programs, or generation of graphical objects. The common view of GE is that, given a particular problem statement, a program that satisfied the fitness function is to be generated. GE is an evolutionary automatic programming type system that uses a combination of a variable length binary string genome and a BNF (Backus-Naur Form) grammar to evolve interesting structures. It presents a unique way of using grammars in the process of automatic programming. Variable-length binary string genomes are used with each codon representing an integer value where codons are consecutive groups of 8 bits. The integer values are used in a mapping function to select an appropriate production rule from the BNF definition, the numbers generated always representing one of the rules that can be used at that time. This technique draws inspiration from the overlapping genes phenomenon exhibited by many bacteria, viruses, and mitochondria that enables them to reuse the same genetic material in the expression of different genes [2].

2.1 Backus-Naur Form

BNF is a notation for expressing the grammar of a language in the form of production rules. BNF grammars consist of terminals, which are items that can appear in the language, e.g., +, -, etc., and nonterminals, which can be expanded into one or more terminals and nonterminals. A grammar can be represented by the tuple \( \{ N \cdot T \cdot P \cdot S \} \), which N is the set of nonterminals, T the set of terminals, P a set of production rules that maps the elements of N to T, and S is a start symbol that is a member of N. When there are a number of productions that can be applied to one particular N, the choice is delimited with the ‘|’ symbol.

Below is an example BNF, where

\[
N = \{ \text{expr}, \text{op}, \text{pre_op} \} \\
T = \{ \text{Sin}, \text{Cos}, +, -, *, /, \text{Variable}, \text{Constant} \} \\
S = <\text{expr}>
\]

And P can be represented as

1. \(<\text{expr}> ::= <\text{expr}>\text{op}<\text{expr}>\ldots\ldots\text{rule 0}\)  \\
   \(| ( <\text{expr}>\text{op}<\text{expr}>).\ldots\ldots\text{rule 1}\)  \\
   \(|<\text{pre-op}> ( <\text{expr}>).\ldots\ldots\text{rule 2}\)  \\
   \(| \text{var}.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldot...
If we assume the codon being read produces the integer 6, then
\[ 6 \mod 4 = 2 \]
would select \(<op>\) as rule 2: /. Each time a production rule has to be selected to map from a nonterminal, another codon is read. In this way, the system traverses the genome. For example, consider the individual in Table 1. There are fourteen 8-bit binary codons in one string. The decoding process is described as follows.

(1) First, concentrating on the start symbol \(<expr>\), we can see that there are four productions to choose from. To make this choice, we read the first codon from the chromosome “11001000” and use it to generate a number “200”. Because the standard decode of the binary 11001000 is
\[
1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0
\]
which equals to 200. This number will then be used to decide which production rule to use according to Eq. (1) in BNF. Thus, we have 200 \(\mod 4 = 0\), meaning we must take the zeroth production, rule (0), so that \(<expr>\) is now replace with \(<expr><op><expr>\).

Table 1. Example of each codon converted into corresponding BNF grammar

<table>
<thead>
<tr>
<th>No</th>
<th>8-bit binary codon</th>
<th>Integer value</th>
<th>Mapping function</th>
<th>BNF grammars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11001000</td>
<td>200</td>
<td>200 (\mod 4 = 0)</td>
<td>(&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>2</td>
<td>10100000</td>
<td>160</td>
<td>160 (\mod 4 = 0)</td>
<td>(&lt;expr&gt;&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>3</td>
<td>11001110</td>
<td>206</td>
<td>206 (\mod 4 = 2)</td>
<td>(&lt;pre-op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>4</td>
<td>01100000</td>
<td>96</td>
<td>96 (\mod 4 = 3)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>5</td>
<td>00011011</td>
<td>27</td>
<td>27 (\mod 4 = 3)</td>
<td>(\sin(&lt;var&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>6</td>
<td>01001000</td>
<td>72</td>
<td>72 (\mod 4 = 0)</td>
<td>(\sin(&lt;var&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>7</td>
<td>00100111</td>
<td>107</td>
<td>107 (\mod 4 = 3)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>8</td>
<td>00111110</td>
<td>62</td>
<td>62 (\mod 4 = 2)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>9</td>
<td>00010110</td>
<td>22</td>
<td>22 (\mod 4 = 2)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>10</td>
<td>00110111</td>
<td>55</td>
<td>55 (\mod 4 = 3)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>11</td>
<td>01100000</td>
<td>88</td>
<td>88 (\mod 4 = 2)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>12</td>
<td>01001100</td>
<td>100</td>
<td>100 (\mod 4 = 0)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>13</td>
<td>11001011</td>
<td>203</td>
<td>203 (\mod 4 = 3)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
<tr>
<td>14</td>
<td>00101001</td>
<td>41</td>
<td>41 (\mod 2 = 1)</td>
<td>(\sin(&lt;expr&gt;)&lt;op&gt;&lt;expr&gt;&lt;op&gt;&lt;expr&gt;)</td>
</tr>
</tbody>
</table>

(2) Continuing with the first \(<expr>\), i.e., always starting from the leftmost nonterminal, a similar choice must be made by reading the next codon value 160 and again using the given formula we get 160 \(\mod 4 = 0\), i.e., rule 0. The leftmost \(<expr>\) will now be replaced with \(<expr><op><expr>\) to give
\[ <expr><op><expr><op><expr><op><expr>. \]
(3) Again, we have the same choice for the first \(<expr>\) by reading the next codon value 206, the result being the application of rule 2 to give
\[ <pre-op><expr><op><expr><op><expr><op><expr>. \]
(4) Now, the leftmost \(<pre-op>\) will be determined by the codon value 96 that gives us rule 0, which is \(<pre-op>\) becomes \(\sin\). We have the following:
\[ \sin(<expr>)<op><expr><op><expr><op><expr>\]
(14) The mapping continues until eventually we are left with the following expression:
\[ \sin(X)\cos(X)+1.0 \]

3. GE Combined with Genetic Algorithm

3.1 Genetic Algorithm

The genetic algorithm (GA) is an iterative procedure, which maintains a population of individuals that are candidate solutions to specific domain. During each generation, the individuals in the current population are rated for their effective evaluations, and a new population of candidate solutions is formed using specific genetic operators such as reproduction, crossover, and mutation. These steps are repeated until the convergence criterion is satisfied or a predetermined number of generations are reached. Reproduction is a process in which individual trees are set according to their fitness function values. The reproduction (selection) operator may be implemented in algorithmic form in a number of ways, such as proportional, rank, tournament selection.

3.2 GE GA

First, a GE was employed to transfer the binary string through BNF grammars to mathematical...
function which maps the input onto output. Further, a GA was incorporated with this GE to optimize the objective value of those functions. In other words, the GA was used as a search strategy to determine the most proper relationship among the observed data.

4. A Case Study of Inflow Prediction in De-Chi Reservoir

In this paper, the proposed GE coupled with traditional MR is applied to the De-Chi Reservoir for inflow forecasting.

4.1 System Description

Located in central Taiwan the Da-Chia river is about 140 kilometers long with an average channel slope of 1/39 and a total watershed size of 1,236 square kilometers. De-Chi Reservoir, which was completed in 1974 and has an efficient storage capacity of 169*10^6 m^3, is one of the major storage reservoirs in Da-Chia River Basin, shown in Figure 1. The primary water use in the basin is hydroelectric power generation, because it is the steepest channel in Taiwan.

![Figure 1. The De-Chi Reservoir and the basin of Da-Chia River in Taiwan](image)

**The Inflow Data**

Historical ten-day (the traditional time period of reservoir operation in Taiwan) inflows to the reservoir for a period of 40 years (1959-1998), excluding three years were used for modeling. Three typical cases of inflow were chosen as the testing cases, case 1-the dry condition (year 1964); case 2-the wet condition (year 1986) and case 3-the normal condition (year 1997).

4.2 Forecast through GE Modeling

This system identification problem may be viewed as a search for a proper function (and its parameters) which maps input values onto an output value. According to the statistical correlation analyses, the first (t-1), second (t-2), third (t-3), and 36th (t-36) ahead ten-day inflow were chosen as the input variables to predict the inflow at time t.

**The Objective Function**

The main consideration of objective function of the inflow prediction model is to minimizing the mean absolute error (MAE). The criterion of MAE is suitable for measuring the accuracy of the whole flow data [3]. It for the ten-day periods is defined as follows:

\[
MAE = \frac{1}{N} \sum_{t=1}^{N} |\hat{Q}_t - Q_t| \quad \text{.........................(2)}
\]

Where
- \( Q_t \): the actual inflow at time t
- \( \hat{Q}_t \): the predicted inflow at time t
- \( N \): the total number of time steps
- \( t \): time steps (ten-day)

4.3 Multi-Regressive Analysis

To compare with traditional multiple regression (MR), the same input variables were used to construct the model, shown as follows.

\[
Q_t = Q_{t-1} \cos(Sin(Q_{t-1}))+\log(Q_{t-1})+\log(Q_{t-1}) \quad \text{............... (3)}
\]

Where
- \( \hat{Q}_t \): the predicted inflow at time step t
- \( Q_{t-1} \): the actual inflow at time step t-1
- \( Q_{t-2} \): the actual inflow at time step t-2
- \( Q_{t-3} \): the actual inflow at time step t-3
- \( Q_{t-36} \): the actual inflow at time step t-36
- \( t \): time step (ten-day)
In order to compare the predicting ability of these two models, four statistical and hydrological indexes were proposed. The variables are defined as follows.

- $tQ$: the actual inflow at time $t$
- $\hat{tQ}$: the predicted inflow at time $t$
- $Q$: the average of actual inflow
- $\bar{Q}$: the average of predicted inflow
- $N$: the total number of time steps
- $t$: time steps (ten-day)

(1) Mean Absolute Error—MAE (defined as equ. (2))

$RMSE = \sqrt{\frac{\sum(Q - \hat{Q})^2}{N}}$ ........................................ (4)

MAE and RMSE are two indexes for calculating the deviations between the actual and predicted value. The smaller value represents the better results. The RMSE is influenced more by higher deviations, whereas the MAE is an unbiased interpreter of the forecast performance [4].

(3) Percentage Absolute Error—PAE

$PRE = 1 - \frac{MAE}{Q}$ ............................................. (5)

(4) Coefficient of Correlation—CC

$CC = \frac{\sum\hat{Q}(Q - \hat{Q})}{\sqrt{\sum\hat{Q}^2(Q - \hat{Q})^2}}$ ........................................ (6)

When the value of PAE and CC approach 1, means the prediction is more accurate.

### 4.5 Results and discussion

The multi-regressive (MR) method is presented for comparison with the newly developed GE. The linear function obtained from MR is shown as the following equation.

$35.840 + 0.470tQ_{-1} + 0.633tQ_{-2} + 0.072tQ_{-3} - 0.122tQ_{-4}$ (7)

With population size =100 and through 800 generations, the final optimal equation obtained from GE is shown as equ.(8). It is indicated that the equation is complex non-linear form, including the terms of Sin, Cos and Log defined in <pre-op>.

$Q_{-1} \times \text{Cos}(\text{Sin}(tQ_{-1})) + \text{Log}(tQ_{-2}) + \text{Log}(tQ_{-3})$ ................................. (8)

In case 1, the dry year 1964, the MAE of GE is 34.512; and that of MR is 105.774. Obviously, the result of GE is much better than that of the traditional MR method. The actual and predicted inflows generated by GE and MR in the year 1964 are shown in Figure 2.

![Figure 2. The inflows predicted by GE and MR on dry year (1964)](image-url)
Table 2. The criteria of GE and MR on dry year (1964)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>GE</th>
<th>MR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>34.512 *</td>
<td>105.774</td>
</tr>
<tr>
<td>RMSE</td>
<td>60.868 *</td>
<td>143.498</td>
</tr>
<tr>
<td>CC</td>
<td>0.923 *</td>
<td>0.724</td>
</tr>
<tr>
<td>PAE</td>
<td>0.859 *</td>
<td>0.568</td>
</tr>
</tbody>
</table>

As in case 2, the wet year 1986, except RMSE of both methods is almost equal; GE outperforms MR on the other three criteria shown in Table 3.

Table 3. The criteria of GE and MR on wet year (1986)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>GE</th>
<th>MR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>130.732 *</td>
<td>153.090</td>
</tr>
<tr>
<td>RMSE</td>
<td>222.792</td>
<td>218.155 *</td>
</tr>
<tr>
<td>CC</td>
<td>0.796 *</td>
<td>0.742</td>
</tr>
<tr>
<td>PAE</td>
<td>0.701 *</td>
<td>0.650</td>
</tr>
</tbody>
</table>

The predicted results of GE and MR versus actual inflow are plotted in Figure 3. Owing to the peak flows are very high in this wet year; both methods are unable to predict them precisely.

The under-estimations at the 9th and 15th time steps and the over-estimations at the 22nd and 25th time steps are however more serious by using MR than GE. Finally, in case 3, the normal year 1997, all these four criteria indicate that the GE is also the better one shown in Table 4. The predicted results of GE and MR are plotted in Figure 4. Therefore, based on the four criteria presented in this paper, GE has the better prediction results than traditional MR on three different hydrological conditions.

Table 4. The criteria of GE and MR on normal year (1997)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>GE</th>
<th>MR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>103.588 *</td>
<td>130.125</td>
</tr>
<tr>
<td>RMSE</td>
<td>229.747 *</td>
<td>231.410</td>
</tr>
<tr>
<td>CC</td>
<td>0.700 *</td>
<td>0.691</td>
</tr>
<tr>
<td>PAE</td>
<td>0.699 *</td>
<td>0.622</td>
</tr>
</tbody>
</table>

The symbol * represents the better one between these two methods in Table 2~4.
5. Conclusions

The grammatical evolution (GE) has been presented to predict the inflow of a reservoir and compared with the conventional multi-regressive (MR) models. Because of the complexities of function types, the capability of fine tuning is better using GE in the case study. It can deal easily with nonlinear transfer problems among several input and output data. Results reported here have shown that GE outperforms traditional multi-regression (MR) on all of the three scenarios, especially for the dry year condition. The results of comparison also indicate that the GE is a powerful tool for input-output mapping and can be effectively used for reservoir inflow forecasting. Further researches can be improved to use the real-coded expression of GE and GA. The coefficients of equation can be more precise and accurate.

References


