$H_{\infty}$ Output Feedback Control for Continuous-time Stochastic T-S Fuzzy System with State-dependent Noise

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Abstract

In this study, we propose a robust $H_{\infty}$ fuzzy control under generalized dynamic output feedback scheme design method for a class of continuous-time nonlinear stochastic systems with state-dependent noise. Based on Takagi and Sugeno (T-S) fuzzy dynamic model, generalized fuzzy controller is developed to achieve the $H_{\infty}$ control system performance by meeting the Hamilton-Jacobi inequality (HJI). However, for reducing the complicated computation, the controller gain matrices can be obtained via solving some related linear matrix inequalities (LMI) instead of the Hamilton-Jacobi inequality (HJI). Simulation study is provided to illustrate my main results.

Keywords: $H_{\infty}$ control, Output feedback, Stochastic T-S fuzzy model

1. Introduction

The automatic control design for static system has been applied for many years. Main topic can be divided into two parts, one is state feedback control design which the system states are available and another is output feedback control design which the system states can not be obtained. For state feedback control design, [1] presents an experiment of planar two-link flexible manipulator to the effector of the manipulator to track desired trajectories and [2], [3] are some applications of output feedback control design. Unfortunately, sometimes model imprecision may come from actual uncertainty about the plant in reality. Hence, the robust control design is proposed to deal with the uncertainty. Meanwhile, fuzzy system which is constructed from human experts by numerical input-output data has been used in many various fields. Furthermore, Takagi and Sugeno (T-S) fuzzy system [4] is a useful technique to approximate various nonlinear systems. Based on T-S fuzzy model with parametric uncertainties, [5] successfully achieve the $H_{\infty}$ control performance for the nonlinear system either state feedback control scheme or fuzzy observer-based output feedback control scheme by using fuzzy linear controller whose gain matrix design problems are parameterized in terms of linear matrix inequalities (LMI) under boundary uncertainty conditions, and [6] study a fuzzy observer-based control scheme mixed suboptimal $H_2$ control performance under a desired $H_{\infty}$ disturbance rejection constraint for the nonlinear dynamic system by settling some eigenvalue problems (EVP).

Recently, many researches for the stochastic system are more and more popular than the static ones. Unfortunately, the states of stochastic system are varied rapidly thus they can not be formulated as usual. [7] introduces some useful conceptions including Itô-type stochastic differential equation to modify the original differential equation and some literatures are proposed in [8]-[10]. In the nonlinear stochastic system with state-dependent noise, the $H_{\infty}$ control performance is guaranteed by solving the Hamilton-Jacobi inequalities (HJI) which can not be obtained easily. In order to simplify complex computation, some investigations like [11] and [12] characterize the design problems as linear matrix inequalities (LMI) instead of Hamilton-Jacobi inequalities (HJI). Compared with previous literatures, we propose a $H_{\infty}$ output feedback control scheme based on the fuzzy approach for continuous-time nonlinear stochastic systems with state-dependent noise in this study. We shall consider the fuzzy generalized dynamic output feedback scheme. It will be shown that the $H_{\infty}$ control system design problem can be attacked by solving matrix inequalities (MI) which correspond to the Hamilton-Jacobi inequality. Moreover, for solving MI problems, we provide LMI-based sufficient conditions for the nonlinear stochastic systems by substituting for variables to obtain the controller gain matrices. Simulation study will be made to guarantee the $H_{\infty}$ robust control performance for the considered nonlinear stochastic systems with state-dependent noise.

The remainder of this thesis is organized as follows. In Section 2, we propose a class of Itô-type stochastic differential equation for continuous-time nonlinear stochastic systems and construct the fuzzy systems. Then we present the main result in Section 3 to attain the $H_{\infty}$ control system performance under the generalized output feedback scheme with available premise variables. Moreover, simulation study
is made to verified the $H_{\infty}$ control system performance. Finally, conclusions are made in Section 5.

2. System Formulation

Consider a class of continuous-time nonlinear stochastic systems represented by Itô-type stochastic differential equation

$$
\begin{align*}
    dx(t) &= (f_1(x(t)) + g_1(x(t))v(t) + p(x(t))u(t))dt \\
    &\quad + (h(x(t)) + l(x(t))u(t))dW(t) \\
    dy(t) &= (f_2(x(t)) + g_2(x(t))v(t))dt
\end{align*}
$$

where $x(t)$ is the system state, $dy(t)$ is the derivative of measurement output signal, $u(t)$ is the control input signal, $W(t)$ is the standard Wiener-Lévy process. We assume that $f_1(x(t)), g_1(x(t)), f_2(x(t)), g_2(x(t)), p(x(t)),$ and $h(x(t))$ as well as $l(x(t))$ are smooth functions with $f_1(0) = g_1(0) = f_2(0) = g_2(0) = p(0) = h(0) = l(0) = 0$. The process $v(t) \in L^2(H)$ is the external disturbance signal and $L^2(H)$ is the Hilbert space which contains any stochastic process $f(t)$ satisfied that

$$
\|f(t)\|_2^2 \triangleq E \left\{ \int_0^\infty f^T(t) f(t) dt \right\} < \infty.
$$

Remark 1: Physically, we only can obtain measurement output signal denoted by $Y(t)$ with mathematical illustration $y(t) = \int_0^t Y(\tau) d\tau$.

Hence, we use (1) to be our continuous-time nonlinear stochastic system.

In order to approximate the original nonlinear stochastic system (1) for further control system design, a T-S stochastic fuzzy dynamic model, which is proposed by Takagi and Sugeno [4], will be used. By the T-S fuzzy model [4], we get the plant rule of the following form:

Plant Rule $i$ for $i = 1, 2, \ldots, L$:

If $\theta_1(t) = \theta_1, \ldots, \theta_j(t)$ is $F_{ij}$, then

$$
\begin{align*}
    dx(t) &= (A_{ij}x(t) + B_{ij1}v(t) + B_{ij2}u(t))dt \\
    dy(t) &= (C_{ij}x(t) + D_{ij1}v(t))dt
\end{align*}
$$

where $\theta_1(t), \theta_2(t), \ldots, \theta_j(t)$ are premise variables, $A_{ij}, B_{ij1}, B_{ij2}, A_{ij1}, A_{ij2}, C_{ij}$ and $D_{ij1}$ are known constant matrices of appropriate dimensions, $F_{ij}$ is the fuzzy set, and $L$ is the number of Fuzzy-IF-Then rules. Then the fuzzy systems with singleton fuzzifier, product inference, and the center average defuzzifier are inferred as follows:

$$
\begin{align*}
    dx(t) &= \sum_{i=1}^L h_i(t) ([A_{ij}x(t) + B_{ij1}v(t) + B_{ij2}u(t)]dt \\
    &\quad + [A_{ij1}x(t) + A_{ij2}u(t)]dW(t)] \\
    dy(t) &= \sum_{i=1}^L h_i(t) ([C_{ij}x(t) + D_{ij1}v(t)]dt
\end{align*}
$$

where

$$
\begin{align*}
    h_i(t) &= \prod_{j=1}^g F_{ij}(\theta_j(t)) \\
    \theta(t) &= [\theta_1^T(t), \theta_2^T(t), \ldots, \theta_j^T(t)]^T \text{ and } F_{ij}(\theta_j(t)) \text{ is the grade of membership of } \theta_j(t) \text{ in } F_{ij}.
\end{align*}
$$

We presume that

$$
\sum_{i=1}^L \mu_i(t) > 0, \text{ for any } \theta(t)
$$

Therefore, we get

$$
\sum_{i=1}^L h_i(t) \geq 0, \text{ for } i = 1, 2, \ldots, L
$$

and

$$
\sum_{i=1}^L h_i(t) = 1.
$$

3. $H_{\infty}$ Generalized Output Feedback Control Design

Based on the fuzzy model (3), we construct a fuzzy output feedback controller to deal with the control system design problem as described by:

Controller Rule $i$ for $i = 1, 2, \ldots, L$:

If $\theta_1(t) = F_{i1}, \ldots, \theta_j(t) = F_{ij}$ then

$$
\begin{align*}
    d\hat{x}(t) &= \hat{A}_{ij}\hat{x}(t)dt + \hat{B}_i dy(t) \\
    u(t) &= \hat{C}_i\hat{x}(t)
\end{align*}
$$

where $\hat{A}_{ij}, \hat{B}_i,$ and $\hat{C}_i$ are the controller matrices for the $i$-th controller rule and $\hat{x}(t)$ is the state of the controller. By the fuzzy systems mentioned above, we can obtain the overall controller as the following form:

$$
\begin{align*}
    d\hat{x}(t) &= \sum_{i=1}^L h_i(t)\{\hat{A}_{ij}\hat{x}(t)dt + \hat{B}_i dy(t)\}
\end{align*}
$$

and

$$
\begin{align*}
    u(t) &= \sum_{i=1}^L h_i(t)\hat{C}_i\hat{x}(t)
\end{align*}
$$

Substituting (9) into the system (3), the Itô-type stochastic differential equation of the system state can be expressed as:

$$
\begin{align*}
    dx(t) &= \sum_{i=1}^L \sum_{j=1}^g h_i(t)h_j(t)\{[A_{ij}x(t) + B_{ij1}v(t) + B_{ij2}u(t)]dt \\
    &\quad + [A_{ij1}x(t) + A_{ij2}u(t)]dW(t)] \\
    + B_{2i}\hat{C}_i\hat{x}(t)dt + [\Delta A_{ij}x(t) + \Delta B_{2i}\hat{C}_i\hat{x}(t)]dW(t)\}
\end{align*}
$$

Similarly, (8) can be rearranged into an appropriate form for the design purpose:

$$
\begin{align*}
    d\hat{x}(t) &= \sum_{i=1}^L \sum_{j=1}^g h_i(t)h_j(t)\{[\hat{A}_{ij}\hat{x}(t) + \hat{B}_iC_{ij}x(t) + \hat{B}_iD_{ij1}v(t)]dt \}
\end{align*}
$$
With (10) and (11), we establish the augmented closed-loop system as follows:

\[
d\tilde{x}(t) = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(\theta(t)) h_j(\theta(t)) \begin{bmatrix} A_i & B_2 \hat{C}_j \\ \hat{B}_1 C_{ij} & \hat{A}_j \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt + \begin{bmatrix} \Delta A_i \\ \Delta B_2 \hat{C}_j \end{bmatrix} x(t) dW(t)
\]

where

\[
\tilde{x}(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad \hat{A}_j = \begin{bmatrix} A_i & B_2 \hat{C}_j \\ \hat{B}_1 C_{ij} & \hat{A}_j \end{bmatrix}, \\

\hat{B}_{ij} = \begin{bmatrix} B_{ij} \\ \hat{B}_{ij} D_{ij} \end{bmatrix}, \quad \hat{D}_{ij} = \begin{bmatrix} \Delta A_i \\ \Delta B_2 \hat{C}_j \end{bmatrix}
\]

The $H_\infty$ output feedback control design problem based on the fuzzy output feedback control is to find the appropriate controller matrices $\hat{A}_j$, $\hat{B}_i$, and $\hat{C}_i$ for $i, j = 1, 2, \ldots, L$, and attenuate the effect of the external disturbance $v(t)$ on the control variable $z(t) = m(\tilde{x}(t))$ in the sense of energy under a prescribed attenuation level $\gamma^2$, such that

\[
\left\| m(\tilde{x}(t)) \right\|^2_{L_2} \leq \gamma^2 \left\| v(t) \right\|^2_{L_2}
\]

where $m(\tilde{x}(t)) = \tilde{x}(t)^2$. In order to establish the $H_\infty$ output feedback control design for the stochastic system (12), we need some lemmas first as below.

**Lemma 1**: For any vectors $a$ and $b$, the following inequality holds for any real number $\gamma$.

\[
a^T b + b^T a \leq \gamma^2 a^T a + \gamma^{-2} b^T b
\]

**Lemma 2**: Let $X_{ij}$ be any matrices and $P = P^T > 0$. Then we have

\[
[ X_{mn} ] \leq \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(\theta(t)) h_j(\theta(t)) X_{ij}^T X_{ij}^T X_{mn}
\]

**Proof**: Due to lack of space, the proof is omitted. Now, we are ready to present our main result.

**Theorem 1**: Consider the nonlinear stochastic system (12) with the controlled variable $z(t) = m(\tilde{x}(t))$. If there exist a positive function $V(\tilde{x}(t)) \in C^2(R^n)$ and $V(0) = 0$ to the following Hamilton-Jacobi inequality (HJI)

\[
\left( \frac{\partial V}{\partial \tilde{x}} \right)^T f + \gamma^2 \left( \frac{\partial V}{\partial \tilde{x}} \right)^T g + \frac{1}{2} h^T \left( \frac{\partial^2 V}{\partial \tilde{x}^2} \right) h + \left\| m(\tilde{x}(t)) \right\|^2 < 0
\]

then the $H_\infty$ control system performance

\[
\left\| m(\tilde{x}(t)) \right\|^2_{L_2} \leq EV(\tilde{x}(0)) + \gamma^2 \left\| v(t) \right\|^2_{L_2}
\]

holds for some $\gamma > 0$ when the initial augmented system state $\tilde{x}(0) \neq 0$ with $v(t) \neq 0$.

**Proof**: By the Itô formula [13], let the stochastic process $\tilde{x}(t)$ be defined by the equation

\[
d\tilde{x}(t) = [f(\tilde{x}(t)) + g(\tilde{x}(t))v(t)] dt + h(\tilde{x}(t)) dW(t)
\]

where $f(\tilde{x})$, $g(\tilde{x})$, and $h(\tilde{x})$ are continuously differentiable in $E_i$ (Euclidean l-space) and $h(\tilde{x}) \neq 0$. Then the generating operator for this process is

\[
L = \frac{\partial}{\partial t} + \frac{\partial}{\partial \tilde{x}} \left[ f(\tilde{x}) + g(\tilde{x}) v(t) \right] + \frac{1}{2} h^T(\tilde{x}) \left( \frac{\partial^2 V}{\partial \tilde{x}^2} \right) h
\]

Suppose $V(\tilde{x}(t)) > 0$ is the Lyapunov function. By using the generating operator (17), we can obtain

\[
EV(\tilde{x}(t)) \leq \frac{1}{2} h^T(\tilde{x}) \left( \frac{\partial^2 V}{\partial \tilde{x}^2} \right) h + \frac{1}{2} \left( \frac{\partial V}{\partial \tilde{x}} \right) v(t)^2 + \frac{1}{2} \left( \frac{\partial V}{\partial \tilde{x}} \right)^T g + \frac{1}{2} h^T \left( \frac{\partial^2 V}{\partial \tilde{x}^2} \right) h + \left\| m(\tilde{x}(t)) \right\|^2
\]

\[
\leq EV(\tilde{x}(0)) + \gamma^2 \left\| v(t) \right\|^2_{L_2}
\]

(19)

Remark 2**: Suppose that $\tilde{x}(0) = 0$. Then the $H_\infty$ control system performance (15) becomes

\[
\left\| m(\tilde{x}(t)) \right\|^2_{L_2} \leq \gamma^2 \left\| v(t) \right\|^2_{L_2}
\]

Note that the closed-loop system in (12) can be expressed in a compact form as follows:

\[
d\tilde{x}(t) = [f(\tilde{x}(t)) + g(\tilde{x}(t))v(t)] dt + h(\tilde{x}(t)) dW(t)
\]

where

\[
f(\tilde{x}(t)) = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(\theta(t)) h_j(\theta(t)) \tilde{A}_{ij} \tilde{x}(t)
\]

\[
g(\tilde{x}(t)) = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(\theta(t)) h_j(\theta(t)) \tilde{B}_{ij}
\]

\[
h(\tilde{x}(t)) = \sum_{i=1}^{L} \sum_{j=1}^{L} h_i(\theta(t)) h_j(\theta(t)) \tilde{D}_{ij} \tilde{x}(t)
\]

Then, by using Theorem 1, the desired $H_\infty$ control system performance of the stochastic system (12) can be guaranteed. However, the controller gain matrices $\tilde{A}_{ij}$, $\tilde{B}_i$, and $\tilde{C}_i$ for $i, j = 1, 2, \ldots, L$, are not easy to obtain by solving the HJI in (14) directly. To avoid complicated computations from solving the HJI, we need to find some effective ways to solve

\[
\text{Proof: By the Itô formula [13], let the stochastic process}
\]

\[
\text{where}
\]

\[
\text{Then the generating operator for this process is}
\]

\[
\text{Suppose}
\]

\[
\text{Consider the desired output control signal}
\]

\[
\text{Then we can obtain}
\]

\[
\text{by Lemma 1}}
\]

\[
\text{by (14)}
\]

\[
\text{Note that the closed-loop system in (12) can be expressed}
\]

\[
\text{where}
\]

\[
\text{Then, by using Theorem 1, the desired}
\]

\[
\text{However, the controller gain matrices}
\]

\[
\text{To avoid complicated computations}
\]

\[
\text{To avoid complicated computations from solving the HJI, we need to find some effective ways to solve}
\]
the matrices $\hat{A}_{ij}$, $\hat{B}_i$ and $\hat{C}_i$. To this end, a condition which can simplify the $H_\infty$ control system design is offered in the following.

**Theorem 2:** For the augmented system (12), if there exists a matrix $P = P^T > 0$, which is the common solution of the following matrix inequalities:

$$
\begin{bmatrix}
\phi_{ij} & PB_{ij} & \tilde{D}_{ij} & I \\
B_{ij}^T P & \frac{1}{4}\gamma^2 I & 0 & 0 \\
D_{ij} & 0 & -P^{-1} & 0 \\
I & 0 & 0 & -I
\end{bmatrix} < 0
$$

(23)

where $\phi_{ij} = (P\hat{A}_{ij} + \hat{A}_{ij}^T P)$, for $i, j = 1, 2, \ldots, L$, then the $H_\infty$ control system performance

$$
\|m(\tilde{x}(t))\|^2_{L_2} \leq E\{\tilde{e}^T(0)P\tilde{e}(0)\} + \gamma^2\|v(t)\|^2_{L_2}
$$

(24)

is guaranteed for some $\gamma > 0$.

**Proof:** Define a Lyapunov candidate function as

$$
V(\tilde{x}) = \tilde{x}^TP\tilde{x}
$$

(25)

where $V(\tilde{x}) \in C^2(R^n)$ and $V(0) = 0$. By Theorem 1 and (12), if any stochastic process can be formed by (21) and inequality (14) holds,

$$
L \sum_{i=1}^L \sum_{j=1}^L h_i(\theta(t))h_j(\theta(t))(\frac{\partial V}{\partial x})^T(\hat{A}_{ij}\tilde{x}) + \gamma^2(\frac{\partial V}{\partial x})^T
$$

$$
\left[\sum_{m=1}^M \sum_{n=1}^M h_m(\theta(t))h_n(\theta(t))\tilde{D}_{mn}\tilde{x}\right] + \frac{1}{2}\sum_{i=1}^L h_i(\theta(t))h_i(\theta(t))\tilde{x}^T\tilde{D}_{ii}\tilde{x} + \frac{1}{2}\tilde{x}^T\tilde{D}_{jj}\tilde{x} + \gamma^2 \tilde{x}^T\tilde{D}_{ij}\tilde{x} < 0
$$

(26)

then the $H_\infty$ control system performance (15) can be concluded. Substituting (25) into (26), we can derive the below inequality

$$
\sum_{i=1}^L \sum_{j=1}^L h_i(\theta(t))h_j(\theta(t))(\tilde{x}^T[P\hat{A}_{ij} + \hat{A}_{ij}^TP + 4\gamma^2 P\hat{B}_{ij}\hat{B}_{ij}^TP + \tilde{D}_{ij}^TP\tilde{D}_{ij} + I]\tilde{x}) < 0
$$

(27)

Accordingly, if the following matrix inequalities

$$
P\hat{A}_{ij} + \hat{A}_{ij}^TP + 4\gamma^2 P\hat{B}_{ij}\hat{B}_{ij}^TP + \tilde{D}_{ij}^TP\tilde{D}_{ij} + I < 0
$$

(28)

hold, for $i, j = 1, 2, \ldots, L$, then the $H_\infty$ control system performance (24) can be guaranteed. By Schur complement [14], the above matrix inequalities (28) can be rewritten as

$$
\begin{bmatrix}
\phi_{ij} & PB_{ij} & \tilde{D}_{ij} & I \\
B_{ij}^T P & \frac{1}{4}\gamma^2 I & 0 & 0 \\
D_{ij} & 0 & -P^{-1} & 0 \\
I & 0 & 0 & -I
\end{bmatrix} < 0
$$

(29)

where $\phi_{ij} = (P\hat{A}_{ij} + \hat{A}_{ij}^TP)$, which imply that (22) holds. Hence, by Theorem 2, we can conclude that the $H_\infty$ control system performance (24) holds.

However, the $H_\infty$ control design problem, which is to find the common solution $P = P^T > 0$, controller gain matrices $\hat{A}_{ij}, \hat{B}_i, \hat{C}_i$ for $i, j = 1, 2, \ldots, L$ from the matrix inequalities (MI) (23), can not be obtained by using the linear matrix inequality (LMI) technique. Therefore, as quoted from [9], we specify the matrix $P$ as the following form:

$$
P = \begin{bmatrix}
X & Y^{-1} - X \\
Y - 1 & X
\end{bmatrix} > 0
$$

(30)

where $X = X^T > 0$ and $Y = Y^T > 0$. Then we propose a theorem to transfer the matrix inequalities (MI) (23) into a sufficient condition under which can be solved by the LMI technique so that the system (12) can possesses the $H_\infty$ control system performance (24).

**Theorem 3:** Consider the augmented system (12). If there exist matrices $X = X^T > 0, Y = Y^T > 0, B_i$, and $C_i, i = 1, 2, \ldots, L$ such that the following linear matrix inequalities are satisfied, then the $H_\infty$ control system performance (24) can be guaranteed.

$$
\begin{bmatrix}
X & I \\
Y & Y
\end{bmatrix} > 0
$$

(31)

where

$$
\begin{bmatrix}
A_{ii} & 0 & B_{ii} \\
0 & A_{22} & XB_{11} + B_{21}D_{11} \\
D_{11}^T & \Delta A_i & 0
\end{bmatrix} > 0
$$

(32)

The matrices in the output feedback controller can be obtained via

$$
\hat{B}_i = (Y^{-1} - X)^{-1}B_i
$$

(34)

$$
\hat{C}_i = C_iY^{-1}
$$

(35)

$$
\hat{A}_{ij} = (Y^{-1} - X)^{-1}M_{ij}Y^{-1}
$$

(36)

where

$$
M_{ij} = -\hat{A}_{ij}^T + 2A_iY - XB_2C_j - B_iC_j
$$

(37)

**Proof:** Based on Theorem 2, if there exists a matrix $P = P^T > 0$ such that matrix inequalities (23) hold, then the
augmented system (12) is guaranteed with the $H_{\infty}$ control system performance (24). For the matrix $P = P^T > 0$ in (30), we need the conditions (31) to ensure $X = X^T > 0$ and $Y = Y^T > 0$. Now suppose there exists a matrix $P = P^T > 0$ such that (23) holds, i.e.,

$$
\begin{bmatrix}
\phi_{ij} & P \tilde{B}_{ij} & \tilde{D}_{ij}^T & I \\
\tilde{B}_{ij}^T P & -\frac{1}{4} \gamma^2 I & 0 & 0 \\
\tilde{D}_{ij} & 0 & -P^{-1} & 0 \\
I & 0 & 0 & -I \\
\end{bmatrix} < 0 \quad (38)
$$

where $\phi_{ij} = (P \tilde{A}_{ij} + \tilde{A}_{ij}^T P)$. To replace the nonlinear term $P^{-1}$ in (38) with a linear upper bound, we consider the following condition:

$$
(F - P^{-1})^T P (F - P^{-1}) = F^T P F - F^T I F + P^{-1} \geq 0 \quad (39)
$$

where $F$ is any matrix with appropriate dimension. By choosing $F = \begin{bmatrix} I & Y \\ 0 & Y \end{bmatrix}$, then (39) can be rewritten as follows:

$$
-P^{-1} \leq \begin{bmatrix} X - 2I & I - Y \\ I - Y & -Y \end{bmatrix} \leq \bar{P} \quad (40)
$$

Based on the above analysis, (38) can be rewritten as the following matrix inequalities (MI):

$$
\begin{bmatrix}
\phi_{ij} & P \tilde{B}_{ij} & \tilde{D}_{ij}^T & I \\
\tilde{B}_{ij}^T P & -\frac{1}{4} \gamma^2 I & 0 & 0 \\
\tilde{D}_{ij} & 0 & \bar{P} & 0 \\
I & 0 & 0 & -I \\
\end{bmatrix} < 0 \quad (41)
$$

By multiplying $\begin{bmatrix} \bar{W}^T & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$ from the left and $\begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix}$ from the right where $\bar{W} = \begin{bmatrix} \bar{W} \phi_{ij} \bar{W}^T & \bar{W} \tilde{B}_{ij} & \bar{W} \tilde{D}_{ij}^T & \bar{W} I \\ \tilde{B}_{ij}^T \bar{W}^T & -\frac{1}{4} \gamma^2 I & 0 & 0 \\ \tilde{D}_{ij}^T \bar{W}^T & 0 & \bar{P} & 0 \\ \bar{W}^T & 0 & 0 & -I \end{bmatrix}$ we obtain

$$
\begin{bmatrix}
\tilde{W} \phi_{ij} \tilde{W}^T & \tilde{W} \tilde{B}_{ij} & \tilde{W} \tilde{D}_{ij}^T & \tilde{W} I \\
\tilde{B}_{ij}^T \tilde{W}^T & -\frac{1}{4} \gamma^2 I & 0 & 0 \\
\tilde{D}_{ij}^T \tilde{W}^T & 0 & \tilde{P} & 0 \\
\tilde{W}^T & 0 & 0 & -I \\
\end{bmatrix} < 0 \quad (42)
$$

Then substituting $\bar{W}$, $\bar{P}$ in (13), (34), (35), and (36) into (42), we can derive

$$
\Phi_{ij} < 0 \quad i, j = 1, 2, \ldots L
$$

where $\Phi_{ij}$ is shown in (33), which imply that (32) holds. Hence, by Theorem 2, we can conclude that the $H_{\infty}$ control system performance (24) holds.

Apparently, (32) are linear matrix inequalities (LMI). Thus variables $X$, $Y$, $B_i$ and $C_j$, for $i, j = 1, 2, \ldots, L$, can be easily obtained, for example, by using the Matlab LMI toolbox. Hence, substituting $X$, $Y$, $B_i$, as well as $C_j$ into (36) and (37), we can get the variable $\tilde{A}_{ij}$. Moreover, using the fact $\tilde{B}_i = (Y^{-1} - X)^{-1} B_i$ and $\tilde{C}_j = C_j Y^{-1}$, we can determine controller gains $\tilde{A}_{ij}$, $\tilde{B}_i$ and $\tilde{C}_j$ to complete the fuzzy controller for achieving $H_{\infty}$ control system performance (24).

According to previous discussion, a design procedure for a class of nonlinear stochastic systems in (1) to attain $H_{\infty}$ control system performance is given below.

step 1): Design a T-S fuzzy system for the nonlinear stochastic system and choose an appropriate initial value of $\gamma^2$.

step 2): Use (31)-(37) to obtain $\tilde{B}_i$ and $\tilde{C}_j$.

step 3): Substitute $\tilde{B}_i$ and $\tilde{C}_j$ into (37) to get $\tilde{A}_{ij}$.

step 4): If $\tilde{B}_i$, $\tilde{C}_j$, and $\tilde{A}_{ij}$ are solvable, then decrease $\gamma^2$ and repeat step 2) - step 3) until $\tilde{B}_i$, $\tilde{C}_j$ and $\tilde{A}_{ij}$ can not be found.

step 5): Construct the fuzzy controller.

4. Simulation Study

In this section, two simulation examples are given to verify the proposed design methods for the considered stochastic system. Now we consider a fuzzy model defined as follows:

Rule (1): If $x_1$ is about $-10$, then

$$
\begin{aligned}
dx(t) &= (A_1 x(t) + B_{11} v(t) + B_{21} u(t)) dt + (\triangle A_1 x(t) + \triangle B_{21} u(t)) dW(t) \\
dy(t) &= (C_{11} x(t) + D_{11} v(t)) dt
\end{aligned}
$$

(43)

Rule (2): If $x_1$ is about 0, then

$$
\begin{aligned}
dx(t) &= (A_2 x(t) + B_{12} v(t) + B_{22} u(t)) dt + (\triangle A_2 x(t) + \triangle B_{22} u(t)) dW(t) \\
dy(t) &= (C_{12} x(t) + D_{12} v(t)) dt
\end{aligned}
$$

(44)

Rule (3): If $x_1$ is about 10, then

$$
\begin{aligned}
dx(t) &= (A_3 x(t) + B_{13} v(t) + B_{23} u(t)) dt + (\triangle A_3 x(t) + \triangle B_{23} u(t)) dW(t) \\
dy(t) &= (C_{13} x(t) + D_{13} v(t)) dt
\end{aligned}
$$

(45)

where

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -3 & 2 \\ -2 & -5 \end{bmatrix}, & A_2 &= \begin{bmatrix} -5 & -1 \\ -1 & -5 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} -3 & -2 \\ 2 & -5 \end{bmatrix}, & B_{11} &= \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix}, & B_{12} &= B_{11} = B_{13}, \\
B_{21} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, & B_{22} &= B_{21} = B_{23}, & C_{11} &= \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \\
C_{12} &= \begin{bmatrix} 1 \\ 3 \end{bmatrix}, & C_{13} &= \begin{bmatrix} 1 \\ 5 \end{bmatrix}, & D_{11} &= 1, & D_{12} &= 1, \\
D_{13} &= 1, & \triangle A_1 &= \begin{bmatrix} -1 \\ 0.2 \end{bmatrix}, & \triangle A_2 &= \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}, \\
\triangle A_3 &= \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, & \triangle A_3 &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \\
\triangle B_{22} &= \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, & \triangle B_{23} &= \begin{bmatrix} -0.1 \\ 0 \end{bmatrix},
\end{aligned}
$$

As aforementioned, $W(t)$ is the standard Wiener-Lévy process with zero mean and unit variance, $v(t)$ is an external
In this paper, we offer a generalized dynamic output feedback scheme design method for a class of continuous-time nonlinear stochastic systems with state-dependent noise to achieve the $H_\infty$ control system performance by solving the Hamilton-Jacobi inequality (HJI). Based on Takagi and Sugeno (T-S) fuzzy model, generalized fuzzy output feedback controller is represented to attain the $H_\infty$ control system performance by solving some related linear matrix inequalities (LMI) instead of the Hamilton-Jacobi inequality (HJI) for reducing the complicated computation. Simulation study is verified our main results.

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